PROJECT TUBEFLIGHT

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FAR - FIELD AERODYNAMICS
OF TUBEFLIGHT PROPULSION
FAR-FIELD AERODYNAMICS OF TUBEFLIGHT PROPULSION

by

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SUMMARY

Tubeflight is a high-speed tube transport scheme in which the vehicle derives its propulsion from the fore-to-aft transfer of air within the tube. The power required for tubeflight propulsion depends not only on the gasdynamics of the transfer flow but also on the amplitude of the disturbances that are generated by the vehicle in the far field, and these in turn depend on the energy conversion efficiency of the propulsion mechanism.

A method is developed for the coupling and solution of the equations governing the flow field for the case of a tubeflight vehicle in steady motion in a very long tube. The method produces useful information on the interrelationships between the speed of travel, the drag of the vehicle, the amplitude of the flow disturbances in the far field, the energy conversion efficiency of the thrust generator, and the power demands.
SYMBOLS

$A_t$  cross-sectional area of the tube

$A_V$  maximum cross-sectional area of the vehicle

$C_D$  drag coefficient (based on $A_V$ and flow conditions at $a$)

$C_f$  friction coefficient

$d_t$  internal diameter of the tube

$F = pA + \mu u = pA(1 + \delta M^2)$ (stream force)

$\overline{F} = F/F_\infty$

$h$  specific static enthalpy

$h_0^*$  specific stagnation enthalpy

$\overline{h}^* = h_0^*/h_\infty^*$

$k$  heat transfer coefficient

$m$  mass flow rate

$M$  Mach number

$N = M (1 + \frac{\gamma-1}{2} M^2)^{\frac{1}{2}}/(1 + \delta M^2)$

$p$  static pressure

$P_r$  Prandtl number

$R$  gas constant

$u$  flow velocity in vehicle-fixed coordinate system

$x$  distance along tube axis in vehicle-fixed coordinate system

$\beta = A_V/A_t$ (blockage ratio)

$\delta$  ratio of specific heats

$\delta$  prescribed value of $\overline{F}_c - \overline{F}_a$

$\eta$  energy conversion efficiency of the thrust generator

$\rho$  air density

Subscripts:

$\infty$  undisturbed flow

$a,b,c$  see Fig. 1
INTRODUCTION

The tubeflight vehicle propels itself by means of an on-board flow induction device that transfers air from the front to the rear of the vehicle within the tube (Ref. 1). This mode of propulsion is called "internal," to distinguish it from the "external" modes of tube transport propulsion, which are those in which the thrust is generated as the reaction to a force exerted on the tube or on some other stationary body. The external modes themselves are conveniently grouped in two categories (Ref. 2): in the first one, which includes wheel traction and linear motor drives, the air in the tube merely acts as a resisting medium, whereas in the second one, which is typified by pneumatic dispatch and its derivatives, the vehicle is driven by air which is pumped through the tube, directly or indirectly, by external means.

Estimates of the power demands of internal propulsion have so far been limited to ideal situations, in which the far field upstream of the vehicle was assumed to be undisturbed (Refs. 1, 3, and 4). It is known, however, that these power demands depend not only on the energy transformations that take place in the transfer passage but also on those that are induced to occur in the far field, and that the latter in turn depend on the energy conversion efficiency of the propulsion mechanism. It is the purpose of the present paper to examine these interrelationships and to evaluate their effects on the power requirements of tubeflight propulsion.

In dealing with a vehicle traveling at constant speed in a tube of infinite length, the flow in the tube can be treated as steady in a vehicle-fixed frame of reference. In early analyses of the far field under these conditions (Refs. 2, 5, and 6), steady-flow solutions could readily be obtained for the supersonic case (i.e., for vehicles traveling
at supersonic speeds). No such solution could be found, however, for the flow behind the vehicle in the subsonic case unless the pressure behind the vehicle was assumed to be everywhere equal to the undisturbed pressure at infinity. This strange result was essentially confirmed by subsequent studies (Refs. 7 through 9), which revealed that in the far field behind a subsonic vehicle the pressure is everywhere so close to the pressure at infinity that it can indeed be assumed to be equal to it. If, then, the vehicle is represented as a discontinuity across which enthalpy and velocity jump by specified amounts, the steady-flow solution is in all subsonic cases one whereby the flow upstream of the vehicle so adjusts itself to the specified jumps that the pressure throughout the far field behind the vehicle is equal to the pressure at infinity (see, e.g., the linearized analysis of Ref. 8).

Analytical methods have been developed in Refs. 7, 10 and 11 for the study of the far-field flow in nonsteady off-design conditions, but their application to such situations has so far been very limited (Ref. 12). Most of the available analyses deal only with internally-propelled vehicles in steady level flight or, exceptionally (Ref. 2), with steady off-design conditions (e.g., the steady motion of a vehicle driven by both internal and external propulsion, or the steady climb or descent of an internally propelled vehicle).

The analysis developed here is similarly limited in scope but differs from previous ones in method and in that it relates more directly the far-field disturbances to the energy conversion efficiency and to the power demands. The method is based on consideration of the changes of stream force and stagnation enthalpy to which the flow relative to the vehicle is subjected within the tube. These changes follow what
might be called a "dynamic cycle" (to avoid confusion with the thermal
cycle to which the same fluid particles are also subjected). The matching
"jump" conditions are explicitly formulated, for each travel Mach
number, in terms of the vehicle drag coefficient and of a suitably de­
efined thrust generator efficiency. The dynamic cycle lends itself to
simple graphical representation. Thus, the approach developed here is
capable of producing an instructive visualization of the interrelationship
between travel speed, body drag, far-field flow disturbances, thrust
generator efficiency, and power demands. It provides, in addition, a
convenient method for the coupling and quantitative solution of the far­
field flow equations. The method is also applicable to steady off-design
conditions.

ANALYSIS OF THE CYCLE

The Analytical Model

The vehicle is simulated by a drag-actuator disc doublet (Fig. 1)
and is stipulated to be in steady motion in a tube of infinite length.
Fig. 1 identifies, in the vehicle-fixed coordinate system, the stations
to which reference will be made by subscripts in the analysis. The far
field ahead of the vehicle extends from \( x = -\infty \) to station \( a \), and the
far field behind the vehicle extends from station \( c \) to \( x = +\infty \).

The external temperature is assumed to be uniform and equal to the
internal undisturbed air temperature \( T_\infty \), and the heat capacity of the
tube wall is assumed to be large enough to make it permissible to neglect
transient wall temperature fluctuations resulting from heat exchanges
with the air inside. Frictional and heat transfer effects in the flow
region between the two discs are neglected.

The entire analysis is carried out in the vehicle-fixed frame of
reference, and the flow is treated as steady, viscous, heat conducting, compressible and one-dimensional throughout.

Only subsonic unchoked transfer flow conditions are considered.

**First Phase -- The Far Field Upstream**

The first phase of the dynamic cycle is the transformation in the far field ahead of the vehicle, from $x = -\infty$ to station $a$.

The flow transformations in the far field are due to friction and heat exchanges with the tube walls. In the selected frame of reference the tube wall moves at the velocity $U_\infty$. Therefore (Ref. 2) the momentum and energy equations for the far-field flow are, respectively,

\[ dF = \frac{1}{2} C_\ell \rho (u_\infty - U) |U_\infty - U| \pi d_t dx \]   \hspace{1cm} (1)

and

\[ \dot{m} \, dh^o = U_\infty dF + \dot{m} \, dq \]   \hspace{1cm} (2)

By the Reynolds analogy with $Pr = 1.0$ (hence $2k = C_\rho$), and since the tube wall temperature is $T_\infty$,

\[ \dot{m} \, dq = k \rho |U_\infty - U| [h_\infty - h - \frac{1}{2} (U_\infty - U)^2] \pi d_t dx \]

\[ = \frac{1}{2} C_\rho \rho |U_\infty - U| [h_\infty^o - h^o - U_\infty (U_\infty - U)] \pi d_t dx \]

Therefore,

\[ \dot{m} \, dq = \left[ \frac{h_\infty^o - h^o}{U_\infty (U_\infty - U)} - 1 \right] U_\infty dF \]

and Eq. 2 becomes

\[ \dot{m} \, dh^o = \frac{h_\infty^o - h^o}{U_\infty - U} dF \] \hspace{1cm} (3)
Eq. 1 shows that \( (dF/dx)/(u_\infty - u) \) is always positive. It follows, then, from Eq. 3, that \( dh^0/dx \geq 0 \) depending on whether \( h^0 \leq h^0_\infty \). Since \( h^0 \) starts at the level \( h^0_\infty \) at \( x = -\infty \), one must conclude that no departure of \( h^0 \) from that level is possible in the far field upstream of the vehicle. Thus, the stagnation enthalpy is constant throughout the upstream flow field.

Accordingly, in Fig. 2 the first phase of the cycle is described by a horizontal (constant stagnation enthalpy) segment extending from a point \( \infty \), representing the undisturbed flow, to a point \( a \) which represents conditions at station \( a \). The value of \( \overline{F_a} \) is, however, still unknown at this stage of the analysis.

Second Phase -- Generation of Drag

The second phase of the cycle occurs through the drag disc, from station \( a \) to station \( b \), and is a Fanno process. The stagnation enthalpy remains constant (still at the level \( h^0_\infty \)), and the stream force decreases by an amount equal to the drag of the vehicle:

\[
F_a - F_b = \frac{1}{2} C_D A_{Vf} \rho_a u_a^2 = \frac{1}{2} C_D \beta F_a \frac{\delta M_a^2}{1 + \delta M_a^2}
\]

The value of \( M_a \) is obtained from the continuity equation in the form \( F_a N_a = F_\infty N_\infty \) (Ref. 13). Finally,

\[
\frac{\overline{F_a} - \overline{F_b}}{C_D \beta} = \frac{1}{2} \overline{F_a} \frac{\delta M_a^2}{1 + \delta M_a^2}
\]

This phase, like the first one, is represented in Fig. 2 by a horizontal segment. The value of \( \overline{F_b} \) depends on the value of \( \overline{F_a} \), and is therefore still unknown at this point. A plot of Eq. 4 is shown in Fig. 3.
Third Phase -- Generation of Thrust

The third phase occurs from station b to station c, through the actuator disc, and is the propulsion phase. Both the stream force and the stagnation enthalpy are increased through the actuator.

The operating condition determines the magnitude and the sign of the difference $F_c - F_a$. If the vehicle is propelled only internally, this difference must be zero in steady level flight, whereas in steady climb or descent it must balance the x-component of the weight of the vehicle.

The relation between $\Delta h^0$ and $\Delta F$ in the propulsion phase depends on the efficiency $\eta$ of the thrust generator. This efficiency is defined as the ratio between the $h^0$ increment that would be required to produce the prescribed $\Delta F$ in the tube flow isentropically and the $\Delta h^0$ that is actually required to produce the same $\Delta F$, starting with the same flow conditions at b. Because of the latter stipulation, $\eta$ is not quite equal to the ratio between the power input that would be demanded by an isentropic thrust generator (the "power required" in the aeronautical sense) and the power input that is actually demanded by the thrust generator on hand for the same flight condition.

By definition, and from the equation of state,

$$F = \dot{m} \left( \frac{\delta - 1}{\delta} \frac{h}{u} + u \right)$$

and

$$h = h^0 - \frac{u^2}{2}$$

Therefore,

$$h^0 = \frac{\delta}{\delta - 1} \frac{F}{\dot{m}} u - \frac{\delta + 1}{\delta - 1} \frac{u^2}{2} \tag{5}$$
Also, for an isentropic transformation in a constant-area duct,

\[
\frac{p}{p_b} = \left( \frac{\rho}{\rho_b} \right)^\delta = \left( \frac{u_b}{u} \right)^\delta
\]

and, again by definition,

\[
\frac{p}{p_b} = \frac{F - \dot{m}u}{F_b - \dot{m}u_b}
\]

Therefore,

\[
F = \dot{m}u - (F_b - \dot{m}u_b) \left( \frac{u_b}{u} \right)^\delta
\]  
(6)

Eqs. 5 and 6 relate \( h^o \) and \( F \) in parametric form, with \( u \) as the parameter. Unfortunately, the relation is rather unwieldy.

Other procedures are available for the determination of the exact functional relationship between \( h^o \) and \( F \) in this process, but none has been found capable of producing a more convenient form of solution.

On the other hand, in the isentropic transformation considered here, \( h^o \) is a well-behaved function of \( F \), and \( (F_c - F_b)/F_b \) and \( (h^o_c - h^o_b)/h^o_b \) are both so small compared to unity that the linearization

\[
\left( h^o_c - h^o_b \right)_s = \left( F_c - F_b \right) \left( \frac{d h^o}{d F} \right)_b
\]  
(7)

(where subscript \( s \) denotes constant entropy) introduces only a very slight error.

In an isentropic transformation, \( dp = \rho dh \). Therefore, for an isentropic flow in a constant-area duct,

\[
dF = A \left( dp + \rho u du \right) = A \rho \left( dh + u du \right) = A \rho dh^o
\]
Thus, Eq. 7 becomes
\[
\left( h_c^o - h_b^o \right)_s = \left( F_c - F_b \right) \frac{u_b}{\dot{m}} \quad (7')
\]

Since \( h_b^o = h_\infty^o \) and \( (h_c^o - h_b^o)_s = \gamma (h_c^o - h_b^o)_s \), and remembering that \( u = aM \) and \( \dot{m} = (\gamma/RT)^{3/2}FN \) (Ref. 13), one finally obtains
\[
(\gamma (h_c^o - l)) = \frac{\gamma - 1}{\gamma N_\infty} M_b \left( 1 + \frac{\gamma - 1}{2} M_b^2 \right)^{-\frac{1}{2}} \left( F_c - F_b \right) \quad (9)
\]

The Mach number \( M_b \) is obtained from the continuity equation \( \dot{F}_b N_b = N_\infty \).

A plot of Eq. 9 is presented in Fig. 4.

This phase of the cycle is described in Fig. 2 by a straight-line segment \( \hat{b} - \hat{c} \), the slope of which is given by Eq. 9. Point \( \hat{c} \) is the intersection of this line with the line describing the closing phase. The latter will be discussed in the following paragraphs.

**Fourth Phase -- The Far Field Downstream**

The differential equations governing the far-field flow behind the vehicle are again Eqs. 1, 2 and 3. In this case, however, \( h^o \) is not constant: since its initial value is higher than \( h_\infty^o \), the stagnation enthalpy must decrease monotonically in the direction of the flow, in accordance with Eq. 3, approaching the undisturbed-flow value asymptotically at infinity downstream.

The function \( u(F, h^o) \), which can be obtained from Eq. 5, is such that Eq. 3 can be integrated only numerically. On the other hand, a very much simpler (although less exact) description of the transformation in this
phase is obtained if, in the light of the already discussed results of Refs. 5 through 9, the static pressure is assumed to be equal to $p$ throughout the far field behind the vehicle. It should be noted that this assumption is equivalent to the linearization

$$h^0 = h^0 + (F - F_{\infty}) \left( \frac{dh^0}{dF} \right)$$

because this is what Eq. 3 becomes when $p = \text{constant}$ and therefore $\Delta F = \dot{m} \Delta u$. Thus, this phase of the dynamic cycle, like the other three, can be represented by a straight line. The slope of this line turns out to be a function of $M_{\infty}$ only. Indeed, for constant pressure and cross-sectional area the equations of state and continuity give $h/u = \text{constant}$, hence $udh = hdu$. Furthermore, $dF = \dot{m} du$. Therefore,

$$u(dh^0 - u du) = (h^0 - \frac{u^2}{2}) du$$

or

$$\frac{dh^0}{dF} = \frac{h^0 + \frac{u^2}{2}}{\dot{m} u}$$

and the slope of the line $c = \infty$ is

$$\left( \frac{dh^0}{dF} \right)_{c = \infty} = \frac{h^0 + \frac{u^2_{\infty}}{2}}{\dot{m} u_{\infty}} \frac{F_{\infty}}{h^0}$$

$$= \frac{1 + \frac{\gamma}{2} M_{\infty}^2}{\gamma M_{\infty}^2} \frac{1 + (\gamma - 1) M_{\infty}^2}{1 + \frac{\gamma - 1}{2} M_{\infty}^2}$$

This relation is plotted in Fig. 5.

Matching of the Phases

The coupling and solution of the equations developed in the preceding
paragraphs can be performed by iterative but very simple graphical or numerical methods for any tube-vehicle-thrust generator combination and for any steady operating condition.

The numerical procedure is preferable in extensive parametric analyses or when, as is often the case, the maximum amplitude of the flow disturbances is very small. On the other hand, the special merit of a graphical construction of the dynamic cycle is that it reveals, at a glance, a great deal of valuable information on the interaction of the various parameters involved and on their individual or combined effects on performance. For this reason, the present discussion will deal primarily with the graphical analysis and with the interpretation of its results.

It is convenient to start the construction of the cycle by drawing the "closing line" through \( \infty \), with the slope given by Eq. 10. Then an arbitrary value is selected for \( \bar{F}_a \), and the corresponding value of \( \bar{F}_b \) (hence the corresponding position of point \( b \)) is determined by means of Eq. 4. Next, the propulsion line, the slope of which is given by Eq. 9, is drawn through point \( b \), and \( \bar{F} \) is measured at its intersection \( c \) with the closing line. The procedure is repeated, starting with new values of \( \bar{F}_a \), until \( \bar{F}_c - \bar{F}_a = \mathcal{C} \). The value of \( h_c^0 \) then determines the power \( \dot{m}_w (h_c^0 - 1) \) that has to be fed to the thrust generator to sustain the specified operating condition.

Qualitative illustrations of the information that can be acquired from the graphical representation of the dynamic cycle are provided by the following examples.

**Examples**

Fig. 6 compares the dynamic cycles for two level-flight situations differing from one another only in the efficiency of the thrust generator.
Cycle $\infty-a-b-c-\infty$ is for $\eta = 1.0$, whereas cycle $\infty-a'-b'-c'-\infty$ is for a lower efficiency. Because of the lower efficiency, the slope of the propulsion line is greater in the latter than in the former cycle. As a consequence, as one would expect, $\bar{h}_c^0 > \bar{h}_c^0$: i.e., a decrease of efficiency results in an increase of the required power input. In addition, however, $F_c' > F_c$ and, since $S = 0$, $F_a' > F_a$. Therefore (see Fig. 3), whereas the amplitude of the flow disturbances in the far field ahead of the vehicle is increased, the drag of the vehicle is decreased somewhat, and so is the amplitude of the flow disturbances downstream of the vehicle. Because of the displacement of point $b$, and of the attendant change of slope of the propulsion line (Fig. 4), the required power input does not vary in inverse proportion to the thrust generator efficiency.

Fig. 7 shows a comparison of dynamic cycles for two situations that are again identical in every respect except one. In this case, the solid-line cycle ($\infty-a-b-c-\infty$) is for a vehicle in steady level flight, whereas the dashed-line cycle ($\infty-a'-b'-c'-\infty$) is for the same vehicle operating with the same thrust generator, at the same speed, and in a tube of the same diameter, but in steady climb. It will be noted that $F_a < F_a$, and that this modification of the flow in the far field upstream has the effect of alleviating the power penalty associated with climb, despite the fact that the aerodynamic drag of the vehicle is higher in the climb condition than in level flight. Thus, the excess power input required to overcome the $x$-component $SF_\infty$ of the weight of the vehicle is less than $SF_\infty u_\infty / \eta$. It can be verified, however, that this result holds true only for high or moderate energy conversion efficiencies, and that if

$$\eta < \frac{(S-1) M_\infty^2}{1 + (S-1) M_\infty^2}$$
the effects are reversed: namely, the drag of the vehicle is lower in climb than in level flight and the excess power input required for climb is higher than the power required to overcome the gravitational effect.

Conversely, as shown in Fig. 8, the power saving in steady descent is less than the gravitational contribution, because of the adverse far-field effects and despite a decrease of drag of the vehicle. And again it can be verified that this result holds true only if the thrust generator efficiency is higher than the value indicated above, whereas the opposite result is obtained for lower efficiencies.

APPLICATION TO TUBEFLIGHT SITUATIONS OF PRACTICAL INTEREST

Some of the effects discussed in the preceding section -- and particularly the dependence of the drag on $\bar{F}_a$, and of the slope of the propulsion line on $\bar{F}_b$, for any given flight Mach number -- are important only when the amplitude of the flow disturbances in the far field is large. When $\bar{F}_a$ and $\bar{F}_b$ are and remain very close to 1.0, these effects are insignificant and the performance analysis can be greatly simplified.

In order to determine whether this simplification is permissible or not when dealing with tubeflight operating conditions of practical interest, two representative situations, involving the same vehicle (the 100-passenger vehicle of Ref. 4)* but two different tube diameters -- 15 ft and 18 ft -- are considered here. The undisturbed state is $T_\infty = 520$ °R, $p_\infty = 2120$ psf, and the flight Mach number is in both cases $M_\infty = .4$ (a Mach number which brings the transfer flow very close to the choking limit in the case of the smaller tube but not in the case of the larger one). The aerodynamic characteristics of the vehicle

* The vehicle diameter and length are 9 ft and 130 ft, respectively, and its weight is 65000 lb.
and of its support system are taken from Ref. 4. Finally, for each case, three operating conditions are considered: steady level flight, steady $5^\circ$ climb, and steady $5^\circ$ descent.

Application of Eqs. 4, 9, and 10 to these situations yields the following results:

**Case I (15 ft-diam. tube)**

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>Level flight</th>
<th>$5^\circ$ Climb</th>
<th>$5^\circ$ Descent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\underline{F_a}$</td>
<td>$\underline{F_b}$</td>
<td>$\underline{F_a}$</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0018</td>
<td>0.9717</td>
<td>0.9902</td>
</tr>
<tr>
<td>0.7</td>
<td>1.0026</td>
<td>0.9725</td>
<td>0.9913</td>
</tr>
<tr>
<td>0.4</td>
<td>1.0045</td>
<td>0.9743</td>
<td>0.9940</td>
</tr>
<tr>
<td>0.1</td>
<td>1.0180</td>
<td>0.9875</td>
<td>1.0129</td>
</tr>
</tbody>
</table>

**Case II (18 ft-diam. tube)**

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>Level flight</th>
<th>$5^\circ$ Climb</th>
<th>$5^\circ$ Descent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\underline{F_a}$</td>
<td>$\underline{F_b}$</td>
<td>$\underline{F_a}$</td>
</tr>
<tr>
<td>1.0</td>
<td>1.000675</td>
<td>0.9894</td>
<td>0.9926</td>
</tr>
<tr>
<td>0.7</td>
<td>1.000965</td>
<td>0.9897</td>
<td>0.9931</td>
</tr>
<tr>
<td>0.4</td>
<td>1.00169</td>
<td>0.9905</td>
<td>0.9944</td>
</tr>
<tr>
<td>0.1</td>
<td>1.00675</td>
<td>0.9955</td>
<td>1.0033</td>
</tr>
</tbody>
</table>

Clearly, $\underline{F_a}$ is so close to 1.0 over this entire range of operating conditions that, in the computation of the required thrust, $M_a$ can safely be assumed to be equal to $M_\infty$ (which is the same as to say that it is valid to assume, as is done in Ref. 4, that the far field ahead of the vehicle is undisturbed).

The assumption $M_b = M_\infty$ (which conveniently reduces the required power input to the product of the required thrust and $\frac{U_\infty}{\eta}$) is similarly, but not quite as generally, justified by the closeness of
It is seen that this assumption, which is also implied in the analysis of Ref. 4, introduces a slight error, on the defect side, in the computation of $u_b$ and, therefore, in the determination of the slope of the propulsion line (see Fig. 4) and of the required power input. The underestimation of $u_b$, while negligible in Case II, can be as high as 4% in some of the level-flight or climb conditions of Case I. It should be noted, however, that the corresponding percentage error in the computation of the power required is somewhat lower, because it is partly balanced by the slight error (always on the excess side) which results from the linearization of the $h^0,F$ relation in the propulsion phase (Eq. 7).

To the extent that the examples discussed above may be viewed as typical of tubeflight situations of practical interest, these results confirm that the far field upstream can be considered undisturbed in performance analyses of such situations, at least so long as the flow relative to the vehicle is steady and not too near choking in the transfer passage.

**ANALYSIS OF ALTERNATIVE TUBEFLIGHT ARRANGEMENTS**

The arrangement considered in the preceding sections -- that of a vehicle with the thrust generator in the rear -- has certain advantages over the reverse or "tractor" arrangement from the standpoints of cabin noise and environment control. On the other hand, a tractor thrust generator is needed in operating conditions involving a choked transfer flow (Refs. 2 and 4). It is of interest, therefore, to examine the effect that the reversal may have on the required power input in the "unchoked" conditions.
Schematics of the reverse arrangement and of the pertinent dynamic cycle (for a case of steady level flight) are shown in Fig. 9. The governing equations for the far-field transformations \((\infty-a\) and \(c-\infty)\) are the same in this situation as in that of the pusher arrangement, whereas those of the near-field processes \((a-b\) and \(b-c)\) are modified by an interchange of subscripts. Thus, for phase \(a-b\), which is now the propulsion phase, one has

\[
\eta \left(h_b^\circ - h_a^\circ\right) = (F_b - F_a) \frac{u_a}{\bar{m}}
\]

or

\[
\eta \left(\bar{h}_b^\circ - 1\right) = \frac{8-1}{8 \bar{N}_\infty} \left(1 + \frac{\bar{r}-1}{2} M_a^2\right)^{-\frac{1}{2}} (\bar{F}_b - \bar{F}_a)
\]

and for \(b-c\), which is now the drag phase,

\[
\bar{h}_c^\circ = \bar{h}_b^\circ
\]

and

\[
F_b - F_c = \frac{1}{2} C_D A_v \rho_b u_b^2
\]

or

\[
\frac{\bar{F}_b - \bar{F}_c}{C_b \beta} = \frac{1}{2} \bar{F}_b \frac{\delta M_b^2}{1 + \delta M_b^2}
\]

Finally, as before, \(\bar{F}_c = \bar{F}_a\).

The primary effect of the reversal is a change of drag, because the thrust generator produces a (positive or negative) increment of dynamic head; and this change will, of course, affect the required power input in accordance with Eq. 12.
Since changes of cross-sectional area in the present analytical model are absent, the dynamic head $\frac{1}{2} \rho u^2$ is everywhere directly proportional to $u$. Therefore, the effect of the thrust generator in front is to increase or decrease the drag of the vehicle depending on whether $u_b \geq u_a$, i.e., or whether $\rho_b \leq \rho_a$, and this in turn depends on the efficiency of the thrust generator -- the lower $\eta$, the lower the ratio $\rho_b / \rho_a$. The cross-over value $\eta_c$ of the efficiency is that which makes $u_b = u_a$. For this condition, denoting the common velocity at $a$ and $b$ by $u_{ab}$, Eq. 5 yields

$$h_b^* - h_a^* = \frac{x}{\gamma - 1} \frac{u_{ab}}{\dot{m}} (F_b - F_a)$$

(5')

and Eq. 11 becomes

$$\eta_c (h_b^* - h_a^*) = \frac{u_{ab}}{\dot{m}} (F_b - F_a)$$

(11')

whence $\eta_c = (\gamma - 1)/\gamma = 0.286$ (for air). Therefore, the thrust generator in front produces a decrease or an increase of drag (and of required power input) depending on whether its efficiency is higher or lower than 28.6%.

The efficiency of the thrust generator is far above this critical value if the vehicle has an all-electric drive but may well be below it if the prime mover is an internal-combustion engine or gas generator. Since noise and environmental control problems are also more severe in the latter case, it may be concluded that the use of this reverse scheme in unchoked-flow operating conditions is potentially advantageous only for electrically-driven vehicles.

For vehicles utilizing an internal-combustion drive, the difficulties just mentioned may be partially circumvented by means of an alternative tractor arrangement which is described in Ref. 14. This arrangement and
its operation are schematically illustrated in Fig. 10. Here a portion of the heat rejected by the prime mover is dumped into the flow downstream of the vehicle, thereby splitting the propulsion phase into two: a high-efficiency phase \( a-b \), in which thrust is generated and the dynamic head of the flow is reduced, and a zero-efficiency phase \( c'-c \), in which the rejected heat is fed into the flow. Despite the presence of the latter phase, the overall required power input would be reduced with this arrangement, because of the lower drag.

CRITIQUE

The results of the preceding analysis raise new questions.

For one thing, they cast doubt on the exact meaning of \( \eta \) as defined in the discussion of the third phase. What is referred to there as "the prescribed\( \Delta F \)" is actually the drag that would have to be overcome if the thrust were generated isentropically. Since it turns out (Fig. 6) that the drag is itself affected by changes of \( \eta \), it must be concluded that the identification of \( \eta \), as defined, with the energy conversion efficiency of the thrust generator is not entirely accurate. Furthermore, and for the same reason, the validity of tube vehicle performance evaluations from the sole standpoint of "power required" in the aeronautical sense must also be called into question.

In accordance with its stipulated limitation to conditions of subsonic unchoked flow (the "subcritical" regimes), the preceding analysis is based on the drag relations that are developed, for these conditions, in Ref. 4. For the calculation of the drag in the supercritical regimes, Refs. 2 and 4 provide a procedure based on consideration of the entropy rise (or, equivalently, of the stagnation pressure loss) in the transfer
passage. Application of the latter procedure to subcritical situations, as was done by this author in Refs. 1 and 3, would appear to be in keeping with the common practice of intake-flow analysis but leads instead to erroneous relations, whereby the stagnation pressure loss and the drag would increase with increasing stream force at the entrance to the transfer passage. The drag-coefficient approach shows that this is the reverse of what actually takes place in the subcritical regimes. The error is insignificant when the far-field disturbances ahead of the vehicle are negligible, as they have indeed been shown to be in most cases of internal propulsion; it may be important, however, in the comparative evaluation of internal and external modes.

In the following discussion, the designation "external propulsion" will be understood to apply only to the modes of the first category. Those of the second category do not lend themselves to a direct comparison with the internal modes (or with the external modes of the first category) because their power demands are functions of a different set of independent variables.

Consider first the case of an externally-propelled vehicle driven by a mechanism that does not add energy to the flow in the frame of reference of the vehicle. In this case, the dynamic cycle is made up of three phases: (1) the far-field transformation upstream of the vehicle, from $-\infty$ to $a$; (2) the near-field transformation from $a$ to $b$, and (3) the far-field transformation downstream of the vehicle, from $b$ to $+\infty$. Phase (1) is isoenergetic as in the case of internal propulsion, and for the same reason. Phase (2) is isoenergetic because of the stipulation concerning the propulsion mechanism. As a consequence, $h^o_b = h^o_\infty$ and, by virtue of Eq. 3, phase (3) is also isoenergetic. Thus, the equations governing the
far-field flow downstream of the vehicle are those of a subsonic Fanno process. Their only solution that satisfies the boundary conditions at infinity downstream is the trivial one of a flow that is uniform and everywhere at rest relative to the walls of the tube. Therefore, $F_b = F_{\infty}$, and $F_a - F_{\infty}$ is equal to the drag of the vehicle. In contrast, Fig. 2 reveals that, in the case of an internally-propelled vehicle with a pusher propeller, $F_a - F_{\infty}$ is smaller or greater than the drag of the vehicle, depending on whether the slope of the propulsion line $b - c$ is smaller or greater than that of the closing line $c - \infty$, i.e., depending on the value of $\eta$ (Eqs. 9 and 10). This means that the drag of the vehicle may be higher or lower with "isoenergetic" external propulsion than with internal propulsion of the pusher variety, depending on the energy conversion efficiency of the latter. This result is at variance with those of Refs. 1 and 3. On the other hand, it can similarly be verified that, when a tractor arrangement is used, the drag of the vehicle is always lower with internal than with isoenergetic external propulsion. The advantage, however, becomes very large only in the supercritical regimes.

In practical situations external propulsion is, of course, never isoenergetic. The dynamic cycle for the case of a wheel-driven vehicle with an internal-combustion powerplant is illustrated qualitatively in Fig. 11. The vehicle is represented by a drag disc, across which the transformation of the flow ($a - b$) is isoenergetic, followed by a Rayleigh transformation in the region from $b$ to $c$, where the heat rejected by the powerplant is fed into the flow. The closing line $c - \infty$ has the same slope as in the case of internal propulsion. The slope of the dashed line $b - d$ is $(F_{\infty}/m_{h\infty})u_{\infty}$ and $a - d$ represents the work done in overcoming
the drag of the vehicle, whereas $b - c$ represents the heat rejected.

The ratio $(\bar{h}_d^0 - \bar{h}_a^0)/(\bar{h}_d^0 - \bar{h}_a^0 + \bar{h}_c^0 - \bar{h}_b^0)$ is the energy conversion efficiency of the drive system. Inspection of the cycle diagram reveals that a decrease of this efficiency will result in a decrease of drag and in an increase of the required power input. A quantitative comparison of dynamic cycles for internally and externally propelled vehicles in sub-critical operation will be presented in a subsequent paper.
REFERENCES


FIG. 1
Steady level flight

Off - design operation

FIG. 2
FIG. 3  Drag of the vehicle
FIG. 4  Slope of line b–c
FIG. 5  Slope of line c–∞
FIG. 6

Steady level flight. Comparison of dynamic cycles for two different energy conversion efficiencies, all other conditions being equal.
Comparison of dynamic cycles for steady level flight ( -a-b-c- ) and for steady climb ( -a'-b'-c'- ) at same speed.
Comparison of dynamic cycles for steady level flight ( -a-b-c- ) and for steady descent ( -a'-b'-c'- ) at same speed.
FIG. 11